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THESIS

OPTIMAL ROUTING OF COORDINATED AIRCRAFT TO IDENTIFY MOVING SURFACE CONTACTS

by

Alvaro Herraiz Solla

June 2017

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13. ABSTRACT

A warship at sea requires awareness of the ships in its vicinity in order to operate safely. This can be a daunting task, even when equipped with multiple shipboard systems. If naval air assets are available, a Tactical Action Officer (TAO) directs them to gain additional information about as many surface Contacts of Interest (COI) as possible. These air asset routes can be inefficient because there are no shipboard systems to aid with route planning. Additional complications include COIs moving during a route and some COIs being more important to visit than others. This thesis formulates and implements two Optimal Routing of Coordinated Aircraft (ORCA) Integer Linear Programs (ILP) to plan air asset routes that visit as many prioritized COIs as possible in a fixed time horizon. We report computation results planning for up to four air assets and up to 80 COIs. Solution time for both ILPs is less than half an hour for typically encountered routing scenarios with less than 40 COIs. For 80 COIs, we find solutions in less than two minutes to visit 61 COIs, or up to 77 if we can wait two hours to obtain the solution.

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OPTIMAL ROUTING OF COORDINATED AIRCRAFT TO IDENTIFY MOVING SURFACE CONTACTS

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ABSTRACT

A warship at sea requires awareness of the ships in its vicinity in order to operate safely. This can be a daunting task, even when equipped with multiple shipboard systems. If naval air assets are available, a Tactical Action Officer (TAO) directs them to gain additional information about as many surface Contacts of Interest (COI) as possible. These air asset routes can be inefficient because there are no shipboard systems to aid with route planning. Additional complications include COIs moving during a route and some COIs being more important to visit than others. This thesis formulates and implements two Optimal Routing of Coordinated Aircraft (ORCA) Integer Linear Programs (ILP) to plan air asset routes that visit as many prioritized COIs as possible in a fixed time horizon. We report computation results planning for up to four air assets and up to 80 COIs. Solution time for both ILPs is less than half an hour for typically encountered routing scenarios with less than 40 COIs. For 80 COIs, we find solutions in less than two minutes to visit 61 COIs, or up to 77 if we can wait two hours to obtain the solution.

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LIST OF ACRONYMS AND ABBREVIATIONS

AIS	Automated Information System
COI	Contact of Interest
FLIR	Forward-Looking Infrared
HCO	Air Asset or Helicopter Controller
ILP	Integer Linear Program
MTSPTW	Multiple TSP with Time Windows
nm	nautical miles
OP	Orienteering Problem
ORCA	Optimal Routing of Coordinated Aircraft
ORCA TI	ORCA using a Time Indexed variable
ORCA VT	ORCA using a Variable for Time
TAO	Tactical Action Officer
TSP	Traveling Salesman Problem
TSPTW	TSP with Time Windows
UAV	unmanned aerial vehicle
VRP	Vehicle Routing Problem
VRPTW	Vehicle Routing Problem with Time Windows

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EXECUTIVE SUMMARY

A warship at sea requires awareness of the ships in its vicinity in order to operate safely. At times, this can be a daunting task, even when equipped with multiple shipboard systems. If naval air assets are available, a Tactical Action Officer (TAO) directs them to gain additional information about as many surface Contacts of Interest (COIs) as possible. These air asset routes can be inefficient because there are no shipboard systems to aid with route planning. Additional complications include COIs moving during a route, and some COIs being more important to visit than others.

This thesis formulates and implements two Optimal Routing of Coordinated Aircraft (ORCA) Integer Linear Programs (ILPs) to plan air asset routes that visit as many prioritized COIs as possible in a fixed time horizon. The ORCA VT formulation uses a continuous time variable to model each COI with multiple nodes, where each node has a time window when it can be visited. The ORCA TI formulation uses binary variables with a finite set of time indexes to construct routes.

There is a rich and extensive body of literature presenting formulations and solution techniques for applications similar to ORCA, but we find no previously published research that directly addresses the application we consider in this thesis. The ORCA VT formulation is an applied case of the Orienteering Problem (OP), while the ORCA TI formulation follows the structure of a time dependent Traveling Salesman Problem (TSP), or a time dependent Vehicle Routing Problem (VRP). Since we consider multiple air assets, it becomes a multiple time dependent TSP if the air assets' endurances are big enough to visit all COIs, or a multiple time dependent VRP application otherwise.

We use a real data set of 80 merchant vessels positions gained during a twelve-hour period in the Strait of Gibraltar to prepare data for testing both ORCA formulations. We calculate nodes positions for all the moving COIs. We consider

up to four air assets to visit the prioritized COIs. All air assets leave and return to the moving mother warship within their endurance. We assume that COIs sail with a constant course and speed far from the coastline, and the mother warship is sailing within an estimated middle position of all the COIs.

We show results for both formulations without heuristics for several sets of different sizes of COIs (20, 40 and 80) and time windows (5, 10, 15 and 20). Thereafter, to speed solution time, we heuristically solve both formulations for one air asset at a time. Finally, we use a heuristic arc elimination, in addition to the looping heuristic, applying it only to the ORCA VT formulation. This heuristic eliminates or penalizes the longest transitions between COIs to reduce the size of the problem.

Due primarily to aircraft speed and range limitations, we estimate the usual mix of helicopters and small UAVs embarked on destroyers and frigates in a typical scenario at sea (far from the coastline) is able to visit approximately 40 COIs during a six-hour time window. Using ORCA to fit manual naval air planning and execution processes implies achieving a solution within 30 minutes. For such typical scenarios, both formulations offer good optimal or close to optimal solutions within this time limit. The ORCA VT uses less air assets than the ORCA TI but it requires a longer time limit, especially if we increase the number of time windows. For unusual big sets of 40 to 80 COIs, we find ORCA TI provides an acceptable solution within 30 minutes, and in some cases we can get feasible solutions in less than two minutes, but ORCA VT with heuristic arc elimination is superior if given up to two hours.

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I. INTRODUCTION

A. PURPOSE AND OVERVIEW

A warship at sea requires awareness of the ships in its vicinity in order to operate safely. At times, this can be a daunting task, even when equipped with multiple shipboard systems. If naval air assets are available, a Tactical Action Officer (TAO) directs them to gain additional information about as many surface Contacts of Interest (COIs) as possible. These air asset routes can be inefficient because there are no shipboard systems to aid with route planning. Additional complications include COIs moving during a route, and some COIs being more important to visit than others. This thesis formulates and implements two Optimal Routing of Coordinated Aircraft (ORCA) Integer Linear Programs (ILPs) to plan air asset routes that visit as many prioritized COIs as possible in a fixed time horizon.

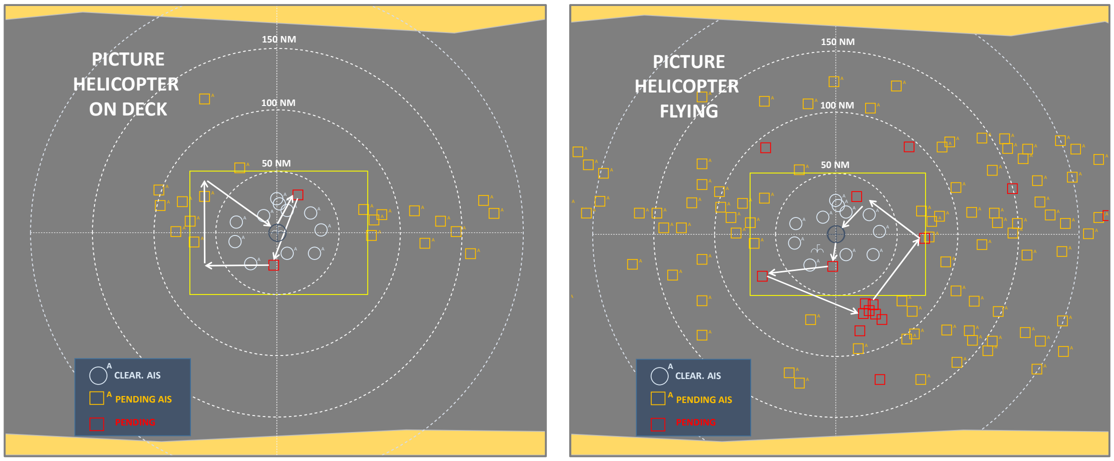
Current warships with helicopter capabilities have shipboard systems that provide a partial surface tactical picture. We define the surface tactical picture as information about ships around a warship displayed in a combat system to help it make tactical decisions. Forward-Looking Infrared (FLIR) imaging systems aboard most warships permit visual identification of all contacts within 12 nautical miles (nm). Some other onboard systems, such as radar systems, enable crews to make relatively accurate deductions within 30 nm. Since 2004, the implementation of the Automatic Information System (AIS) by the International Maritime Organization (2002) allows crews to recognize many contacts within 50 nm by direct signal reception, simplifying and greatly enriching the surface tactical picture, although its range continuously varies with the propagation conditions of the atmosphere, and deception is possible without additional confirmation gained, for example, by an air asset.

Complicating the surface tactical picture, AIS usage is only mandatory for some ships. Usually, all maritime traffic subject to the 1974 International

Convention for the Safety of Life at Sea employs it unless certain conditions, such as a piracy risk, are present in their transiting area. A correlation between remote databases and AIS data received onboard can be made for each contact, achieving an almost complete picture of a huge area. The absence of AIS signals on a radar echo is, in most cases, classified as a COI for surface identification purposes. Many fishing vessels, small boats, and other special ships (like warships) do not transmit AIS signals. The known pattern of sea life allows us to make an educated guess regarding the activities of some of these non-AIS-transmitting ships, and, when the opportunity arises, evaluate them as candidates for further investigation with an air asset.

Warships use air assets—typically a naval helicopter or an unmanned aerial vehicle (UAV)—to complete the surface tactical picture. A surface search flight of any organic air asset—belonging to a destroyer or frigate—is typically conducted twice a day during peacetime operations. In a pre-flight briefing, the TAO, helped by the Air Asset or Helicopter Controller (HCO), establishes the intended flight pattern to be carried out by the pilots, as well as search priorities. Because a surface picture is dynamic, priorities sometimes change after the air asset is underway.

An AIS data equipped air asset can provide useful advantages. When an SH-60 helicopter takes off from deck and gains altitude, the helicopter´s AIS picture can reach as far as 300 nm and this can be shared with its warship using a data link. Figure 1 presents a hypothetical example to show how different the picture can become with an airborne air asset. Therefore, some gaps in the information provided by the AIS equipment onboard the warship are instantly completed, and the surface tactical picture snapshot changes. Then, the desired route for the helicopter can change as well.



Without an Equipped Air Asset Flying (left). With an Equipped Air Asset Flying (right).

Figure 1. Hypothetical Example of the Surface Tactical Picture.

Figure 2 shows a TAO and HCO working environment in a warship's Combat Information Center. Figure 3 shows common types of air assets employed to identify surface COIs.

Source: Department of the Navy (2017) (left); Source: Commander, U.S. Pacific Fleet (2017) (right).

Figure 2. Shore-Based Aegis Ashore Team Trainer in Virginia Beach, VA (left). Console Operator Aboard USS *Normandy* (CG 60) (right).

Source: Armada Española (n.d.) (left); Source: Nieto (2016) (right).

Figure 3. Spanish Navy SH-60B Helicopter during In-flight Refueling (left). Scan Eagle Takeoff from ESPS *Galicia* (L-51) (right).

B. AIR ASSETS

Various embarked air assets operate from warships and are employed to investigate COIs. This thesis considers AIS and FLIR equipped helicopters, and longer endurance UAVs.

1. SH-60

The SH-60 is a naval helicopter (see Figure 4) widely used by many navies around the world since the 1970s. It is manufactured by United Technologies Corporation, Sikorsky Aircraft Division, and is currently in service in many of its variants and evolutions.

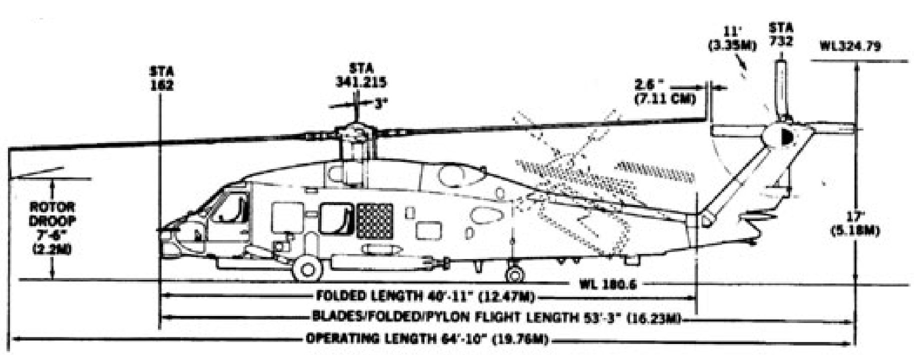


Figure 4. SH-60 Silhouette. Source: Sikorsky Archives (2017).

2. Scan Eagle

The UAV Scan Eagle (see Figure 5) is currently in use in the U.S. Navy. It is manufactured by Insitu Inc., and has proved to be an efficient and low-cost alternative for Identification, Surveillance, and Reconnaissance also in the maritime environment. Table 1 shows data from several open sources, highlighting the assumed endurance and the assumed speed of the UAV, as well as its maximum range of operation from the ship.

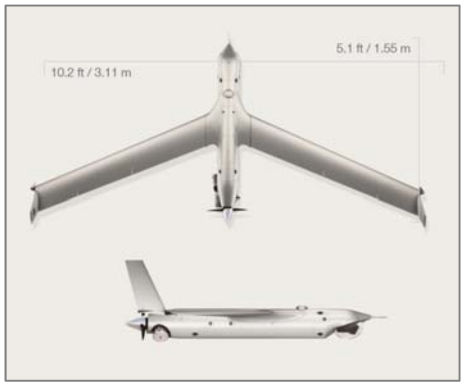


Figure 5. Scan Eagle UAV. Source: Insitu (n.d.).

Table 1. Air Assets Data.

Parameter	SH-60	Scan Eagle
Max Endurance (hours)	3.5	24+
Assumed Endurance (hours)	**3**	**6**
Service Ceiling (feet)	12.000	15.000
Max Speed (knots)	180	80
Cruise speed (knots)	140	50-60
Assumed Speed(knots)	**120**	**60**
Range (nautical miles)	190	80

Adapted from Department of the Navy (2017), Lockheed Martin (n.d.), and Insitu (n.d.).

C. CONTACTS OF INTEREST

A vessel's designation as a COI depends on its position and behavior. Most of the time transiting ships follow a precise route to save fuel and time. Many websites track the majority of the AIS vessel positions and density graphs (see Figure 6) show their expected routes. We expect other vessels in designated areas to occasionally loiter due to fishing.

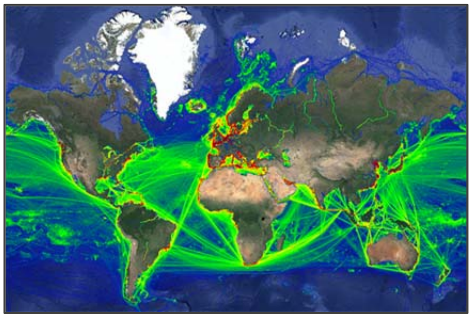


Figure 6. Shipping Density Map. Source: Marine Traffic (n.d.).

COIs move at different speeds, usually from 8 to 25 knots, and in different courses, from 0 to 360 degrees. The mother warship of the air assets is also moving. Figure 7 shows a basic example of this relative motion for several ships in a fixed time interval. Each cell has a 10 nm side. k = 1 represents the mother warship, and k from 2 to 5 are four COIs located near the corners of the graph.

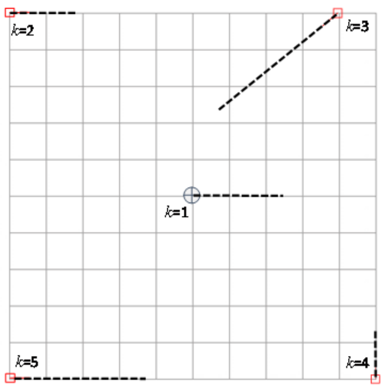


Figure 7. Basic Example of Relative Motion of Five Ships.

Prior intelligence and current operations determine COI priorities. Sometimes, we have intelligence regarding a COI requiring prioritized investigation. It is also important for the surface tactical picture to clarify the identity of any foreign warships in the area. COIs' classification criteria are usually established as a part of the operational task orders, either standing or operation specific. Maritime situation indicators define specific behaviors that should be investigated.

D. MOTIVATION OF THE PROBLEM

Modern combat systems, like AEGIS, offer decision aids employing doctrine to simplify planning, but no such aid exists for planning air routes to investigate as many prioritized COIs as possible. Calfee (2003) shows the power of these doctrine techniques (Auto-Standard Missile, Auto-Special Missile, Identification Friend or Foe, Identification, and Drop-Track doctrines), which permit the operators to simplify their jobs by configuring the system to carry out automatic actions. As a result, some pre-planned reactions are easily and quickly executed with a lower human margin of error.

Personal experience shows that no doctrine or automatic tool exists onboard warships to help the TAO, the Anti-Surface Warfare Officer or the HCO to solve this problem. Many times, the resulting route is poor. Automatic routing of air assets would help busy operators.

E. THESIS ORGANIZATION

Chapter II presents related literature. Chapter III details two different ORCA Mixed Integer Linear Programs (ILPs). Chapter IV explains the data generation. Chapter V proposes constraints to reduce the number of arcs, and Chapter VI presents a real world case study taken from the Strait of Gibraltar. Chapter VII provides conclusions and opportunities for the future research.

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II. LITERATURE REVIEW

There is a rich and extensive literature presenting formulations and solution techniques for applications similar to ORCA, but we find no previously published research that directly addresses the application we consider in this thesis. This chapter reviews select literature that provides formulations similar to those we employ for ORCA. It also reviews some related applications

Dantzig, Fulkerson, and Johnson (1954) describe and obtain a solution to a Travelling Salesman Problem (TSP) with 49 cities in the United States, showing the feasibility of solving problems with a moderate number of nodes using binary variables to denote a transit from city i to city j. Tsiligirides (1984) develops heuristic methods to solve the generalized Orienteering Problem (OP). Golden, Levy and Vohra (1987) propose an effective center of gravity heuristic that improves previous literature, and Balas (1989) formulates the Prize Collecting Traveling Salesman Problem using rewards and penalties for cities visited. Toth and Vigo (2001) describe the TSP with Time Windows (TSPTW) as a special case of the Vehicle Routing Problem with Time Windows (VRPTW) as NP-hard in the strong sense, as well as the Multiple TSPTW (MTSPTW).

Vansteenwegen, Souffriau, and Oudheusden (2010) review the literature of the OP and its applications, presenting the relevant variants with clarity and simplicity. These are similar to the first (time variable) ORCA formulation presented in this thesis.

Lawler, Lenstra, Rinnoy Kan, and Shmoys (1990) summarize the Time Dependent TSP introduced by Fox, Gavish, and Graves in 1980. These are most similar to the second ORCA formulation presented in this thesis. Brown and Carlyle (2008) optimize the employment of U.S. Navy’s Combat Logistics Force, day by day in a whole world scenario given the positions of a number of tasked Battle Groups are known. The base network they employ is scenario independent, with 102 commonly used nodes (as choke points) connected by

198 arcs that can include two arcs between the same node pair representing slow and fast transit. Then, they increment the network in two steps, first adding daily positions of Battle Groups (commonly 13) increasing to around 600 nodes and 700 arcs, and, finally, introducing nodes as waypoints in arc intersections. With around 900 nodes and 64,000 arcs, this simplifying approach enables quick solution time.

In other applications related to ORCA, Sposato (1995) plans “Optimal Routing of Ice Reconnaissance Aircraft” using an OP formulation. The plan seeks routes to identify moving icebergs in a search area discretized into 2,600 nodes. Hartman (2015), develops a Rapid Airlift Planning for Amphibious-Ready Groups Route Optimizing Program as a multiple Vehicle Routing Problem (VRP) with multiple locations, employing earliest to latest admissible landing times of aircrafts that transport personnel and materiel between ships in fixed positions.

There is a related and extensive search theory literature. For example, Stone, Royset, and Washburn (2016) show methods to search moving targets. They present algorithms to find targets both in discrete and continuous time and space. Unlike these search problems, this thesis assumes that all COIs’ positions, constant courses, and constant speeds are available as data, and the main purpose here is to obtain routes for air assets that visit as many prioritized COIs as possible. Although sometimes these kind of missions receive the name of *surface search*, they are not really searching but identifying routes.

III. FORMULATION: THE MIXED-INTEGER LINEAR PROGRAMS

A. INTRODUCTION

This chapter presents two ORCA ILPs along with assumptions, data, variables and constraints for both.

B. TIME WINDOWS AND A VARIABLE FOR TIME OF VISIT

In our first formulation, "VT," we use a variable to represent the visit time for each air asset and COI, and we model each COI using multiple nodes. Each node has a time window in which it can be visited, and we restrict all air assets to visit at most one of the nodes for each COI. In Figure 8, we show the division of the future positions of a COI into time windows based on its course, its speed, and the total endurance of the air asset.

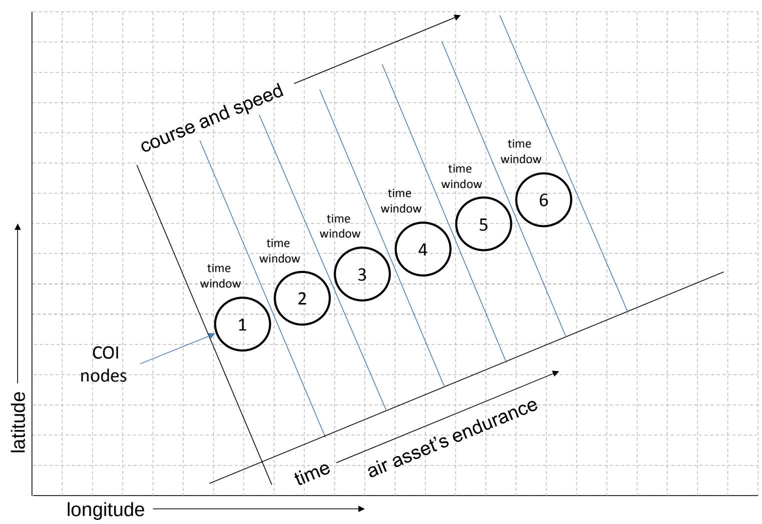

Figure 8. COI Nodes (with 6 Time Windows) Based on Its Course, Its Speed and Air Asset's Endurance.

In Figure 9, we show a small example with 6 time windows per COI and a single air asset at 100 knots with 3.5 hours endurance. If all COIs and mother warship (ship where air assets leave and return) proceed at 10 knots, the side of

each cell is 10 nm, and all COIs' scoring value is the same, the air asset leaves the mother warship, and completes the tour indicated by the blue arrows arriving back at its mother warship in its last time window. Each time window node has an earliest and a latest time of visit. A variable records the time the air asset visits a node.

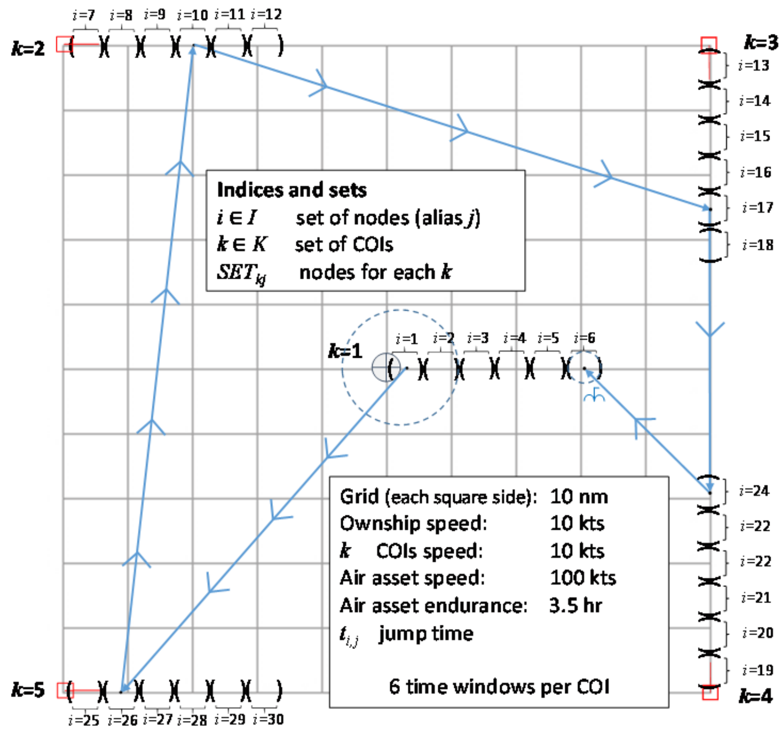

Figure 9. VT Small Example for a Single Air Asset.

1. Assumptions

Air assets routing requires assumptions about flight operations:

1. All air assets take off at the same time.
2. Different types of air assets have different endurance and speed. All air assets have constant speed without considering wind influence, altitude changes, or course variations.
3. All distances are calculated using a great circle formula.
4. It is beneficial to leave an aircraft idle if it is not needed. To model this, we include a prize in the objective function that rewards any air

asset transition from the mother warship's initial position directly to its last position.

5. There is one node for each time window for each COI.

2. Sets

$h \in H$	air assets
$i \in I$	nodes (alias j and p)
$k \in K$	COIs, and the warship ($k = 1$ is warship)
$i \in C_h$	nodes that can be visited by air asset h
$i \in SET_k$	nodes corresponding to COI k
$i \in W_h$	nodes for feasible termination of air asset h route at the mother warship

3. Parameters

e_i , l_i	earliest and latest time to visit i
M	big constant
$nouse_h$	reward for keeping h aboard; $nouse_h \le {}^{1}\!/_{2} \cdot \min\{score_{ih}\}$
$score_{i,h}$	reward for air asset h visiting COI k at node $i \in SET_k$
$\tau_{i,j,h}$	time to fly from i to j for air asset h, such that $e_i + \tau_{i,j,h} \le l_j, \ \forall i, j$

4. Calculated Sets

$i \in IN_j$	all nodes that can immediately precede node j in a feasible tour, such that $e_i + \tau_{i,j,h} \le l_j, \ \forall i, j$

$j \in OUT_i$ — nodes that can immediately follow node i in a feasible tour such that,

$$e_i + \tau_{i,j,h} \le l_j, \ \forall i, j$$

5. **Variables**

$T_{i,h}$ — time of visit to node i by air asset h

$X_{i,j,h}$ — 1 if air asset h goes directly from node i to node j, 0 otherwise

$Y_{i,h}$ — 1 if node i is visited by air asset h, 0 otherwise

6. **Formulation**

$$\max \ \sum_{i,h} score_{i,h} \cdot Y_{i,h} + \sum_{h \ , \ i \in W_h} nouse_h \cdot X_{1,i,h} \tag{3.1}$$

$$\sum_{i \in SET_1 \ , \ j \in OUT_i} X_{i,j,h} \le 1 \qquad \forall h \tag{3.2}$$

$$\sum_{i \in SET_k, \ j \in OUT_i \ , \ h} X_{i,j,h} \le 1 \qquad \forall k > 1 \tag{3.3}$$

$$\sum_{j \in OUT_1} X_{1,j,h} = 1 \qquad \forall h \tag{3.4}$$

$$\sum_{i, \ j \in W_h \bigcap OUT_i} X_{i,j,h} = 1 \qquad \forall h \tag{3.5}$$

$$T_{i,h} + \tau_{i,j,h} \ \le \ T_{j,h} + M\left(1 - X_{i,j,h}\right) \qquad \forall i, j, h \tag{3.6}$$

$$e_i \le T_{i,h} \qquad \forall i \ , h \tag{3.7}$$

$$T_{i,h} \le l_i \qquad \forall i \ , h \tag{3.8}$$

$$\sum_{i \in IN_p \backslash W_h} X_{i,p,h} = \sum_{j \in OUT_p \backslash \{1\}} X_{p,j,h} \qquad \forall h, p \in C_h \tag{3.9}$$

$$\sum_{i \in IN_p} X_{i,p,h} = Y_{p,h} \qquad \forall h, p \notin W_h \ , p \ne 1 \tag{3.10}$$

$$\sum_{j \neq 1} X_{p,j,h} = Y_{p,h} \qquad \forall h, p \notin W_h \ , p \neq 1 \qquad (3.11)$$

$$\sum_{i \in SET_k \ , h} Y_{i,h} \leq 1 \qquad \forall \ k \neq 1 \qquad (3.12)$$

$$X_{i,j,h} \in \{0,1\} \qquad \forall i, j, h$$

$$Y_{i,h} \in \{0,1\} \qquad \forall i, h$$

$$T_{i,h} \in \mathbb{R}^{+} \qquad \forall i, h$$

7. Discussion

The objective function (3.1) expresses the total score obtained by the complete set of air assets with a reward for any unused air asset. Constraints (3.2) ensure that each air asset h leaves the mother warship at most once. Constraints (3.3) guarantee that only one of the air assets can leave a COI k, except the mother warship. Each constraint (3.4) and (3.5) forces all air assets h to begin leaving the mother warship, and to finish arriving the mother warship. Constraints (3.6) provide the time of visit to a node on a route. Each constraint (3.7) and (3.8) enforce the earliest and latest visit times for each node. Constraints (3.9) control the balance of flow for each air asset's type (helicopters and UAVs). Constraints (3.10) and (3.11) ensure an air asset h both enters and leaves a visited node. Constraints (3.12) restrict to one the number of visits to any COI.

C. TIME INDEXED FORMULATION

In our second formulation, "TI," we model each COI using multiple time steps by including a time index on the binary variables. In Figure 10, we show a small example with 6 time steps per COI and a single air asset with 5.5 hours' endurance. For simplicity, we assume only integer time steps for this example. All parameters are similar to the VT formulation example. Binary variables need to be indexed both by the leaving and the arriving node as well as by the proper

time indices for the leaving and arriving time step at each one of the nodes (COIs).

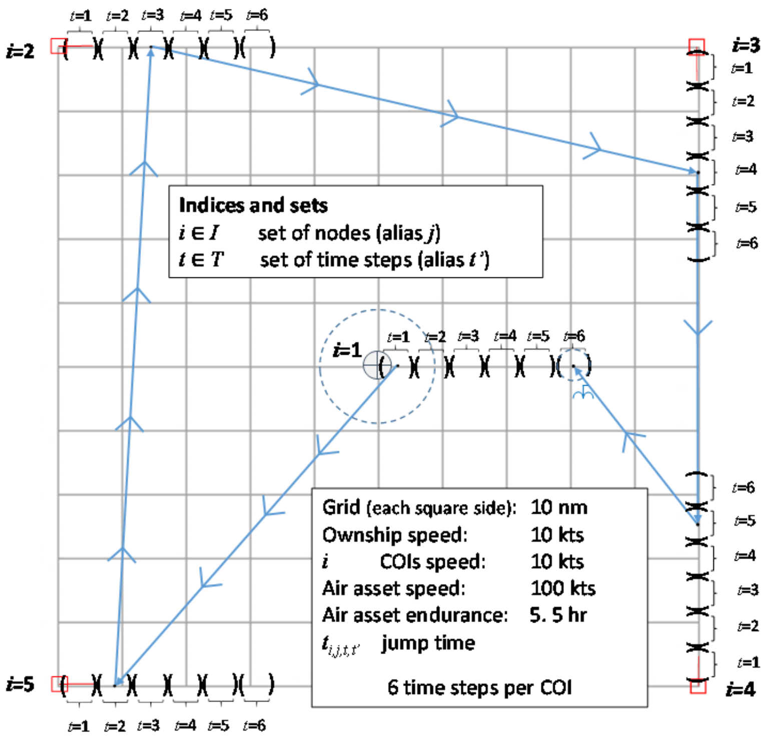

Figure 10. TI Small Example for a Single Air Asset.

1. Assumptions

Assumptions 1 to 4 of the VT formulation are also valid in this formulation.

2. Sets

$h \in H$ air assets

$i \in I$ air assets' mother warship ($i = 1$) and COIs (alias j and p)

$t \in T$ time steps (alias t'), an integer set for all COIs i

$t \in F_h$ set of allowed first times for air asset h

$t' \in L_h$ set of allowed last times for air asset h

3. Parameters

$e_{i,t}$, $l_{i,t}$ — earliest and latest time to visit i at time step t,

$nouse_h$ — reward for keeping h aboard; $nouse_h \leq \frac{1}{2} \cdot \min\{score_{i,t,h}\}, \ \forall i,t$

$score_{i,t,h}$ — score for visiting COI i at time step t with air asset h;

$\tau_{i,j,t,t',h}$ — time to fly from i to j leaving at t and arriving at t' by asset h

4. Calculated Set

$(i,j,t,t',h) \in A$ — set of all feasible transitions for air asset h: $(i,j,t,t',h) \in A$ if

$e_{i,t} + (t'-t) \leq l_{j,t'}$,

$t'-t \geq \tau_{i,j,t,t',h}$,

$i \neq j$,

$(i=1 \wedge t \in F_h) \vee (i \neq 1 \wedge t \notin F_h)$,

$(i=1 \Leftrightarrow t \in F_h)$, and

$(j=1 \Leftrightarrow t' \in L_h)$

5. Variables

$X_{i,j,t,t',h}$ — 1 if air asset h goes directly from COI i at time step t to COI j at time t', 0 otherwise

$Y_{i,t,h}$ — 1 if air asset h visits COI i at time step t, 0 otherwise

6. Formulation

$$\max \sum_{j,\,t',\,h} score_{j,t',h} \cdot Y_{j,t',h} + \sum_{h,\,t \in F_h,\,t' \in L_h} nouse_h \cdot X_{1,1,t,t',h} \tag{3.13}$$

$$\sum_{\substack{j,\,t\,\in F_h,\,t' \mid \\ (1,j,t,t',h) \in A}} X_{1,j,t,t',h} = 1 \qquad \forall h \tag{3.14}$$

$$\sum_{\substack{i,\,t,\,t'\in L_h \,|\\ (i,1,t,t',h)\in A}} X_{i,1,t,t',h} = 1 \qquad \forall h \qquad (3.15)$$

$$\sum_{\substack{i,\,t'\,|\\ (i,p,t',t,h)\in A}} X_{i,p,t',t,h} = \sum_{\substack{j,\,t'\,|\\ (p,j,t,t',h)\in A}} X_{p,j,t,t',h} \qquad \forall p\neq 1,\ \forall t\notin F_h,\ t\notin L_h\ ,\forall h \qquad (3.16)$$

$$\sum_{\substack{i,\,t,\,t'\,|\\ (i,p,t',t,h)\in A}} X_{i,p,t,t',h} = \sum_t Y_{p,t,h} \qquad \forall p,h \qquad (3.17)$$

$$\sum_{\substack{j,\,t,\,t'\,|\\ (p,j,t,t',h)\in A}} X_{p,j,t,t',h} = \sum_t Y_{p,t,h} \qquad \forall p,h \qquad (3.18)$$

$$\sum_{t,h} Y_{p,t,h} \leq 1 \qquad \forall p\neq 1 \qquad (3.19)$$

$$X_{i,j,t,t',h} \in \{0,1\} \qquad \forall (i,j,t,t',h)\in A$$

$$Y_{i,t,h} \in \{0,1\} \qquad \forall i,t,h$$

7. Discussion

The objective function (3.13) maximizes the total score obtained by the complete set of air assets h plus a reward if we do not use all air assets. Constraints (3.14) ensure that all air assets h begin leaving the mother warship during their first time step, while constraints (3.15) ensure that all air assets h finish at the mother warship in the last time step. Constraints (3.16) define the balance of flow for each COI, while constraints (3.17), (3.18), and (3.19) define a single visit to any COI.

IV. DATA PREPARATION

A. INTRODUCTION

ORCA prescribes a routing of air assets given initially defined positions of a number of COIs and their expected future positions. Multisensor combat systems have these data available at all times. To simulate this data, we use a real data set of merchant vessels transiting across the Strait of Gibraltar. This chapter describes this data and its preparation for both formulations.

B. AIS RAW DATA CLEARING

AIS data is easily available and provides data for ORCA test instances. The Spanish Navy´s “*Centro de Operaciones y Vigilancia de Acción Marítima*” (Surveillance and Maritime Action Operations Center), which monitors maritime traffic in the area of interest, provided AIS data taken from the Strait of Gibraltar shore station on November 13, 2016, from 0000Z to 1200Z. Data consists of 80 merchant vessels, which transited across the area during the period. We consider instances with subsets of these 80 COIs as well as all 80.

Figure 11 shows a sample of the raw data as provided in “csv” format. Data for each COI consist of a variable number of lines corresponding to reported positions, as a function of range to the shore station and speed of each COI (i.e., *Nexoe Maersk*, in Figures 11 and 12, with flag of Denmark and Maritime Mobile Service Identity number 219955000, reports 6 position lines from 0218Z to 0250Z, with the first line having timestamp 2016–11-13 at 02:50:05Z, latitude 36’ 01.23,” longitude 005’ 04.53” W, on course 082.5 degrees at 20 knots).

We process the data for each COI using Python 2.7.13 (2017). We assume the last recorded course is constant, and we use the great circle distance to calculate any future position. We locate the mother warship at the mean latitude and the mean longitude of all COIs to simulate data generated by systems on the mothership, at 0400Z (initial reference time for all COIs).

```
Ship Name: NEXOE MAERSK,,,,,
Call Sign: OVYB2,,,,,
Timestamp,Lat,Lon,Heading,Speed,Source
2016-11-13 02:50:05Z,36°01.23'N,005°04.53'W,82.5,20,T-AIS
2016-11-13 02:43:58Z,36°00.97'N,005°07.05'W,83.4,20.4,T-AIS
2016-11-13 02:36:58Z,36°00.72'N,005°09.96'W,85.5,20.3,T-AIS
2016-11-13 02:30:55Z,36°00.54'N,005°12.43'W,84.9,20.8,T-AIS
2016-11-13 02:24:40Z,36°00.37'N,005°15.17'W,87.4,20.6,T-AIS
2016-11-13 02:18:22Z,36°00.27'N,005°17.71'W,87.2,18.5,T-AIS
```

Figure 11. COI Data Example.

Figure 12. *Nexoe Maersk*. Source: Ship Spotting (n.d.).

C. TIME WINDOWS GENERATION

Future positions of moving COIs with constant course and speed can be easily calculated using great circle distances. We consider a time horizon of up to six hours. Each time window is of equal duration and we construct up to 20 time windows (a duration of 18 minutes) for each COI. We consider the COI is located at a fixed position (the mean location during the time window) for the duration of the time window.

Using Google Earth Pro version 7.1.7.2606 (2016), Figure 13 shows all positions associated with all COIs considering six hours and ten time windows. Green and red dots represent first and last positions for the 80 COIs, and yellow

dots represent middle positions. The distribution of all COIs seems close to the real density distribution of the common maritime traffic pattern in the area.

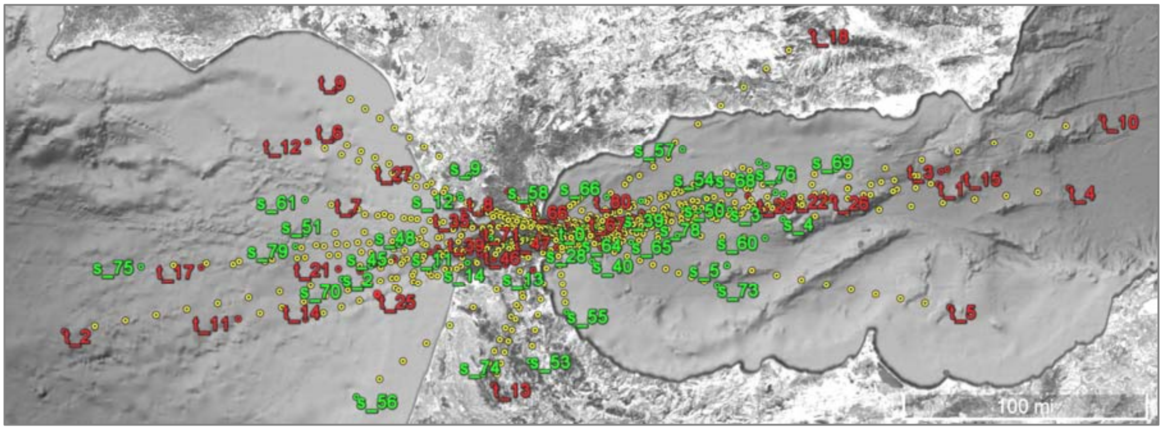

Figure 13. Ten Time Windows for All COIs.

D. SCENARIO RELOCATION

The ORCA problem would typically be used by warships and their air assets located away from coastline chokepoints. To simulate this environment, we relocate all COI positions by introducing an offset of 4 degrees to the west to all latitudes and longitudes. In Figures 14 and 15 we show a transformation of all COI positions to the west.

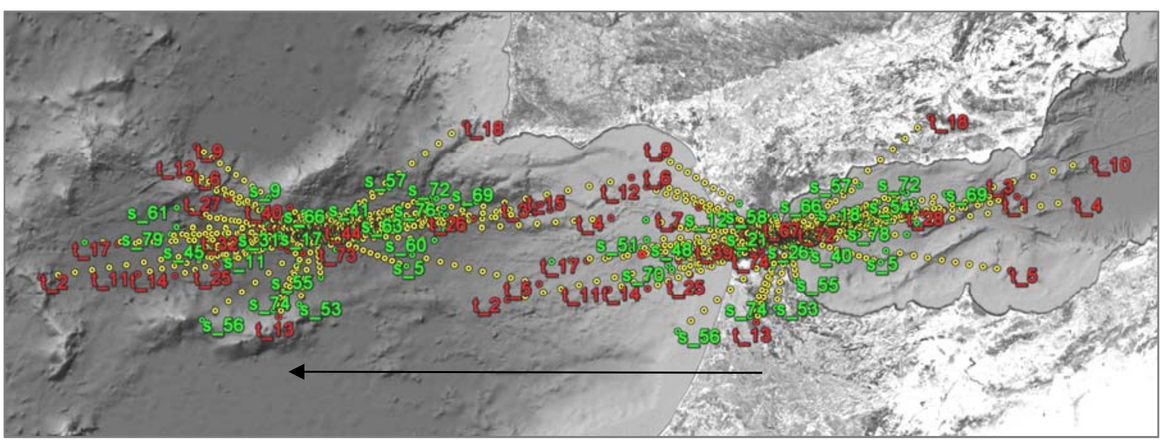

Figure 14. Offset Included for All COIs.

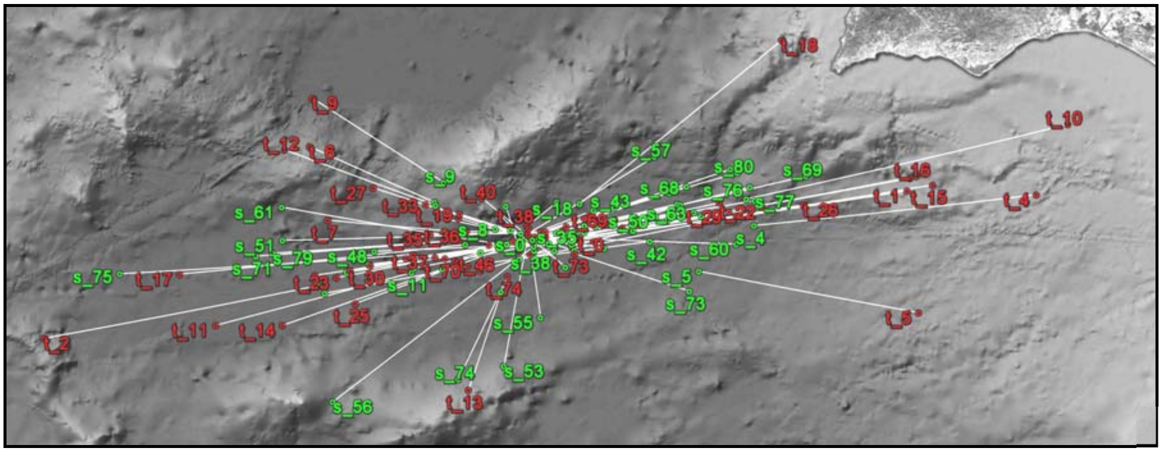

Figure 15. Final Segments of Positions for All COIs.

In Table 2 we show final data after all modifications.

Table 2. Processed COIs Data Used for Chapter VI.

Latitude			Longitude			Course	Speed	Score	#	Name
degs	mins	secs	degs	mins	secs	(degs)	(knts)			
36	2	2	-5	-13	-45	090.0	3.0	100	0	mother warship
36	7	35	-4	-1	-55	082.7	13.5	100	1	N/A
35	46	26	-6	-39	-34	257.6	18.7	100	2	*Cala Pino*
36	9	19	-4	-3	-41	081.0	14.0	100	3	*Thun Goliath*
36	5	52	-3	-42	-31	085.5	18.9	100	4	*YM Wellhead*
35	51	19	-4	-5	-21	102.0	14.8	100	5	*X-PRESS Mulhacen*
36	14	12	-5	-52	-35	293.3	9.0	100	6	*Brussels*
36	5	0	-5	-48	-18	274.6	7.7	100	7	*Lake Ontario*
36	5	42	-5	-28	-3	286.0	2.5	100	8	*British Emissary*
36	20	37	-5	-48	-26	302.1	10.3	100	9	*Irenes Rainbow*
36	14	28	-3	-46	-40	078.7	22.0	100	10	*Rhapsody*
35	51	22	-6	0	-51	254.0	13.4	100	11	*MSC Loretta*
36	13	6	-5	-51	-27	289.5	10.6	100	12	*Marchicora*
35	45	30	-5	-25	-41	198.2	6.8	100	13	N/A
35	52	47	-5	-48	-4	249.9	11.2	100	14	*Stolt Kingfisher*
36	4	15	-4	-35	-56	082.5	20.0	10,000	15	*Nexoe Maersk*
36	4	25	-4	-41	-19	080.8	19.8	100	16	*Aquamarine Ace*
36	1	2	-5	-35	-4	263.2	19.0	100	17	*MSC Fillippa*
36	11	36	-4	-56	-55	051.7	17.1	100	18	*Al Andalus*
36	5	5	-5	-21	0	287.6	3.5	100	19	*Star Omicron*
36	0	35	-5	-24	-29	259.9	9.8	1,000	20	*Arklow Meadow*

Latitude			Longitude			Course	Speed	Score	#	Name
degs	mins	secs	degs	mins	secs	(degs)	(knts)			
36	1	6	-5	-19	-8	260.3	12.8	100	21	*Saga Sapphire*
36	0	6	-5	-10	-37	081.0	14.2	100	22	*Delta IOS*
36	2	14	-5	-13	-38	258.4	12.3	100	23	*Hansa Cloppenburg*
36	4	14	-5	-15	-59	270.0	4.7	100	24	*Krania*
36	3	48	-5	-10	-34	249.0	12.1	10,000	25	*Emerald*
35	58	56	-5	-18	-40	081.9	17.9	100	26	*MSC Ariane*
36	0	47	-5	-6	-32	287.5	11.3	100	27	*SFL Spey*
35	54	25	-5	-23	-35	079.2	14.5	100	28	*Smeraldo*
35	58	46	-5	-22	-27	080.8	13.8	100	29	*Reggedijk*
36	4	18	-4	-52	-24	261.0	13.1	1,000	30	*Aquabreeze*
35	56	34	-5	-41	-11	078.6	16.0	100	31	*STENAWECO Excellence*
36	4	14	-4	-49	-9	262.0	11.3	100	32	N/A
35	57	16	-4	-50	-48	287.2	10.2	100	33	*Azamanta*
35	59	44	-4	-52	-23	281.5	8.9	100	34	*BW Raven*
36	4	40	-4	-46	-12	269.2	10.1	100	35	*Nissos Serifos*
36	4	24	-4	-59	-53	271.0	5.9	100	36	*Mila*
36	7	36	-4	-46	-17	258.0	10.0	100	37	*Ionic Smyrni*
35	57	27	-5	-11	-3	316.0	3.4	100	38	*High Beam*
36	7	22	-4	-41	-17	258.0	10.1	100	39	*Atlantis Aldabra*
35	50	57	-4	-56	-16	309.6	7.1	100	40	*Thorco Isabella*
36	13	24	-4	-42	-34	249.2	9.5	100	41	*Patron*
36	1	39	-4	-17	-8	262.7	11.3	100	42	*Voornedijk*
36	14	23	-4	-33	0	251.7	11.0	100	43	*Herbania*
35	51	6	-6	-9	-26	079.0	13.9	100	44	*Pannonia G*
35	49	9	-6	-41	-2	080.7	19.4	100	45	*MSC Lisbon*
36	15	18	-4	-2	-58	252.0	13.8	100	46	*Blue Ocean*
36	9	23	-4	-23	-31	260.5	9.8	100	47	*Equinox Dawn*
35	57	28	-6	-27	-14	087.2	14.4	100	48	*Brook Trout*
35	52	28	-6	-29	-23	082.6	14.0	100	49	N/A
36	10	36	-4	-14	-50	260.0	10.4	1,000	50	*Atalandi*
36	0	50	-7	-8	-50	087.6	17.3	100	51	*Majestic Maersk*
36	19	48	-4	-6	-57	252.9	11.0	100	52	*STI San*
35	14	30	-5	-25	-49	010.3	9.2	100	53	*Cafer Dede*
36	19	11	-4	-8	-40	253.2	10.3	100	54	*Meridiaan*
35	32	51	-5	-8	-57	351.8	5.5	10,000	55	*Spirit of*
35	0	45	-6	-44	-29	051.9	17.3	100	56	*Opal Leader*
36	32	47	-4	-15	-41	239.0	9.8	100	57	*Cape Cee*
36	14	50	-5	-24	-39	163.4	3.0	100	58	*Navin Eagle*
36	9	7	-4	-10	-2	261.4	9.0	100	59	*Arklow Rover*
35	59	40	-3	-38	-45	272.7	12.5	1,000	60	*Clipper Star*

Latitude			Longitude			Course	Speed	Score	#	Name
degs	mins	secs	degs	mins	secs	(degs)	(knts)			
36	13	37	-7	-8	-49	099.1	15.8	100	61	N/A
36	10	3	-4	-8	-42	264.0	8.6	100	62	*Sealand New*
36	11	43	-3	-58	-15	261.3	9.9	100	63	*Waaldijk*
35	58	11	-5	-2	-25	262.1	1.6	100	64	*Bosporusdiep*
35	58	43	-4	-39	-41	279.9	4.6	10,000	65	*BSLE Genova*
36	15	49	-5	-1	-10	231.6	2.5	100	66	*Lone Star*
36	14	51	-4	-6	-14	257.6	7.7	100	67	*Antari*
36	21	15	-4	-1	-58	252.1	8.3	100	68	*Neptune Kefalonia*
36	24	21	-3	-7	-21	258.0	12.9	100	69	*Cap Felix*
35	41	58	-6	-47	0	078.7	9.6	100	70	*Amavisti*
35	54	54	-7	-19	-1	084.8	13.4	100	71	N/A
36	27	24	-3	-41	-58	252.0	10.1	100	72	*Lada*
35	42	40	-4	-1	-32	288.6	7.9	100	73	*Gremio*
35	10	29	-5	-46	-8	025.4	7.5	100	74	*Laima Uno*
35	47	10	-8	-21	-36	085.1	19.5	100	75	*CMA CGM*
36	20	40	-3	-33	-26	256.5	9.8	100	76	*Atlantic Moon*
36	15	36	-3	-31	-25	261.2	9.7	100	77	*Umar 1*
36	5	8	-4	-27	-41	256.7	4.1	100	78	*Paquito Moreno*
35	56	9	-7	-11	-14	085.3	12.2	100	79	*Ramform Tethys*
36	25	57	-3	-38	-45	254.7	10.1	100	80	*Maersk Denver*

This table consists of all COI's latitude and longitude at the initial reference time, COI's course, COI's speed, scoring obtained for visiting that COI, reference number in the list, and COI's name. This complete list of COIs corresponds to all vessels transmitting AIS signal in the Strait of Gibraltar on November 13, 2016, starting at midnight and ending 12 hours later.

E. TUNING THE SCORING PARAMETER

We need to define the scoring parameter. In Figure 16 we summarize the scoring for route planning. We establish three basic levels of scoring defined as High, Medium, and Low priority, which captures the most common process based on personal experience. Note that maritime situation indicators are present at all times, being mostly used when COI AIS data is not available. AIS discrepancy arises when COI AIS data does not match with behavior, such as heading opposite direction to the port of destination, or other data incoherencies with data bases.

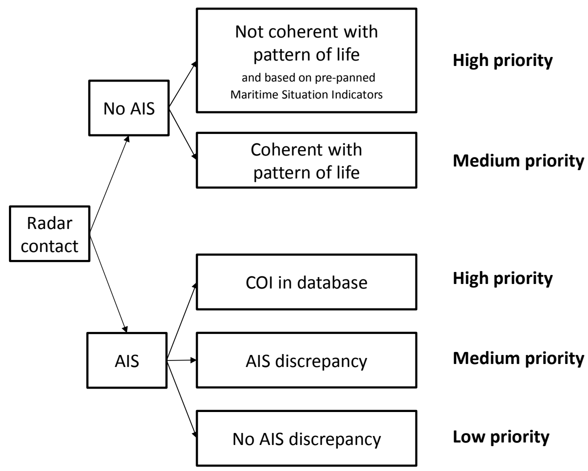

Figure 16. Rewarding Structure.

We modify the score of each COI as a decreasing function in time because it is desired to visit the most important COIs at the beginning of the route. The formula for ORCA VT is the following:

$$score_{i,h} = base_{i,h} - \left(\frac{w}{100 \cdot \left(seq_{i,k} + 1 \right)} \right),$$

where $base_{i,h}$ is fixed for each node based on its priority, $seq_{i,k}$ is the ordinal position of node i in COI k, and w is the total number of time windows. The formula for ORCA TI is the following:

$$score_{i,t,h} = base_{i,1,h} - \left(\frac{w}{100 \cdot seq_t} \right),$$

where seq_t is the ordinal position of time step $t \in T$, and w equals $|T|$, as defined in the ORCA TI formulation. The $base_{i,h}$ and $base_{i,1,h}$ values respond to the priority of each COI and is shown in Table 2. We arbitrarily select four high priority COIs (numbers 15, 25, 55, and 65 in Table 2) with an initial base scoring parameter of 10,000 points. We arbitrarily select four medium priority COIs (numbers 20, 30, 50, and 60) with an initial base score parameter of 1,000 points. Any other COI in the list has an initial base scoring of 100 points. Because it is better to visit any COI with the manned helicopter, we only give half of any COI score value to the UAVs, denoting the preference for helicopters when they are available.

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V. ELIMINATING ARCS

A. INTRODUCTION

This chapter develops constraints to eliminate transitions or combinations of transitions between locations in either model for any asset's route. Its inclusion may eliminate arcs that could be part of an optimal route for an asset. As such, it should be considered a heuristic. That said, it is motivated by some properties of TSP tours that we develop in the following text.

B. CONVERTING A TOUR INTO A CIRCLE

Dantzig et al. (1954) show an optimal tour to a TSP that minimizes distance traveled between 49 cities taking road distances from an atlas (Figure 17). The tour visits all cities exactly once with no subtours.

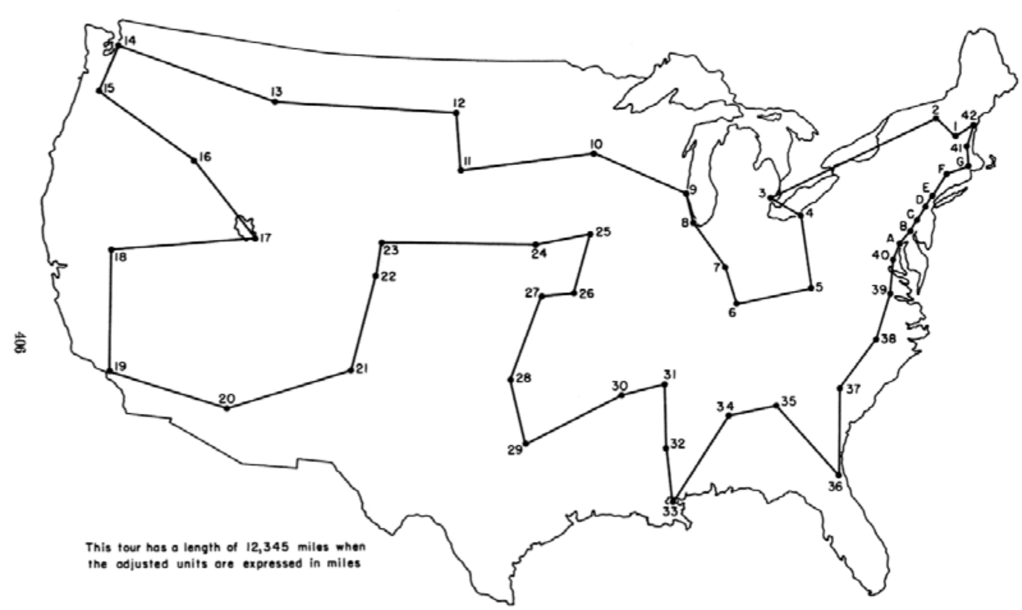


FIG. 16. The optimal tour of 49 cities.

Figure 17. The Optimal Tour of 49 Cities. Source: Dantzig et al. (1954).

We use the optimal tour identified by Dantzig et al. (1954) as motivation. Consider the following sketch (Figure 18) and assume:

- all nodes are located into a 2-dimensional plane;
- all distances are straight lines (a Euclidean TSP); and
- all nodes are connected as in a TSP optimal solution.

Given an optimal tour, we can place nodes on the circumference of a circle such that there is a node for each node of the TSP, and the nodes are ordered such that the distance between two adjacent nodes is the same as in the optimal tour (Figure 18). We call this circle the mass circle. Let us define the mass radius (r_{mass}) as the radius of the mass circle. Then, we can apply the geometry expressed in Figure 19.

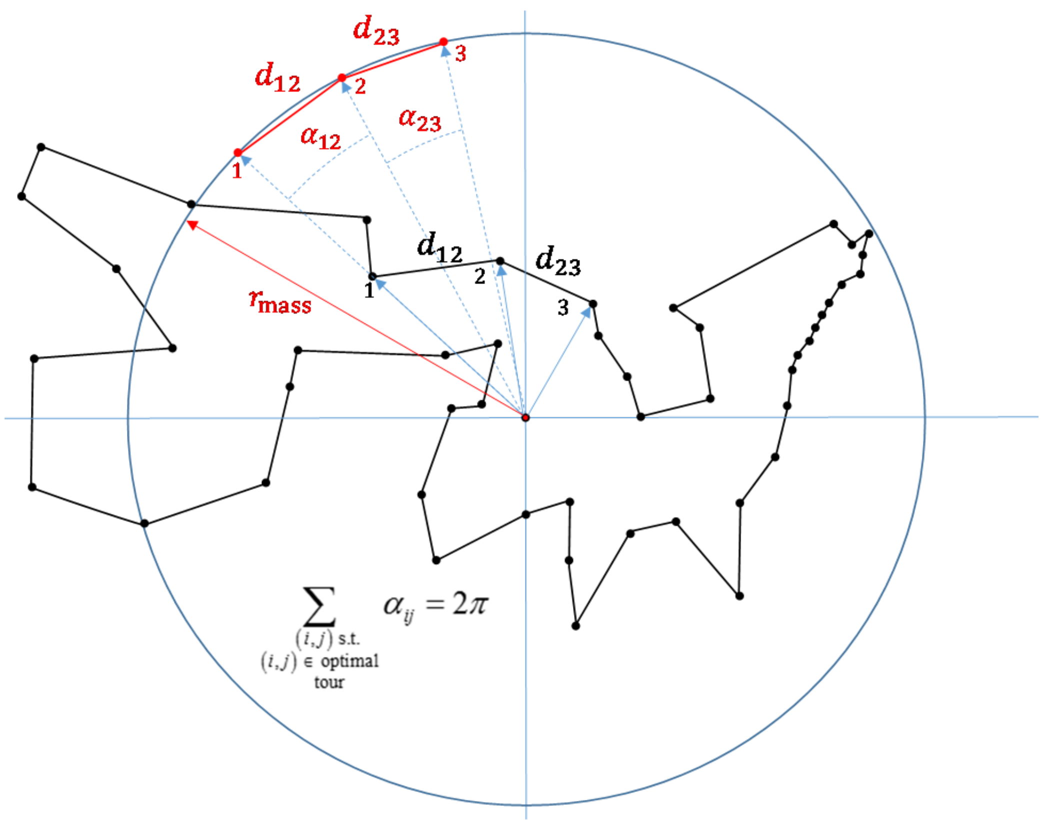


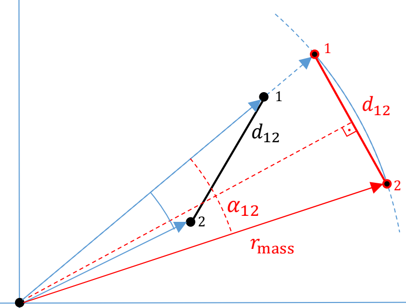
Figure 18. Relocation of a Tour into a Circle.

(4.1) $$\frac{d_{12}}{2} = r_{mass} \sin\left(\frac{\alpha_{12}}{2}\right)$$

(4.2) $$\sin\left(\frac{\alpha_{12}}{2}\right) = \frac{d_{12}}{2 \cdot r_{mass}}$$

(4.3) $$\frac{\alpha_{12}}{2} = \arcsin\left(\frac{d_{12}}{2 \cdot r_{mass}}\right)$$

(4.4) $$\alpha_{12} = 2 \cdot \arcsin\left(\frac{d_{12}}{2 \cdot r_{mass}}\right)$$



(4.5) $$\sum_{\substack{(i,j)\text{ s.t.} \\ (i,j)\, \in \text{ optimal} \\ \text{tour}}} \alpha_{ij} = 2\pi$$

$$\sum_{\substack{(i,j)\text{ s.t.} \\ (i,j)\, \in \text{ optimal} \\ \text{tour}}} \alpha_{ij} = \sum_{\substack{(i,j)\text{ s.t.} \\ (i,j)\, \in \text{ optimal} \\ \text{tour}}} 2 \cdot \arcsin\left(\frac{d_{ij}}{2 \cdot r_{mass}}\right) \quad (4.6)$$

$$\sum_{\substack{(i,j)\text{ s.t.} \\ (i,j)\, \in \text{ optimal} \\ \text{tour}}} \arcsin\left(\frac{d_{ij}}{2 \cdot r_{mass}}\right) = \pi \quad (4.7)$$

Figure 19. Arc Relocation into the Mass Circle.

In equation (4.1), we do a simple trigonometric derivation using the *sine* function. Equations (4.2), (4.3), and (4.4) follow by algebra. Equation (4.5) is obvious since the tour is complete. Following the same logic in all segments as in the case of d_{12}, by addition, we obtain equation (4.6). By algebra and substitution, equation (4.7) follows.

1. Computational Benefits

Given any feasible tour, we can calculate r_{mass} for this tour, and eliminate any arc where $(d_{ij} \,/\, 2\ r_{mass}) > 1$ (exceeds diameter of the circle) because $arcsin(x)$ does not exist for any $x > 1$, and $r^*_{\ mass} \leq r_{mass}$ (where $r^*_{\ mass}$ is the radius for an optimal tour). As an example, see the black irregular pentagon (Figure 20). If we relocate the nodes in a circle, in the same order and same distances between consecutive nodes, we obtain the blue tour, and we see that the radius of the circle is $r_{mass} = 5.34$ (by approximation). Distance $d_{2,4} = 11$ (in the original tour), and $(d_{24} \,/\, 2\ r_{mass}) > 1$. Therefore, its arcsine does not exist, and it can be disregarded.

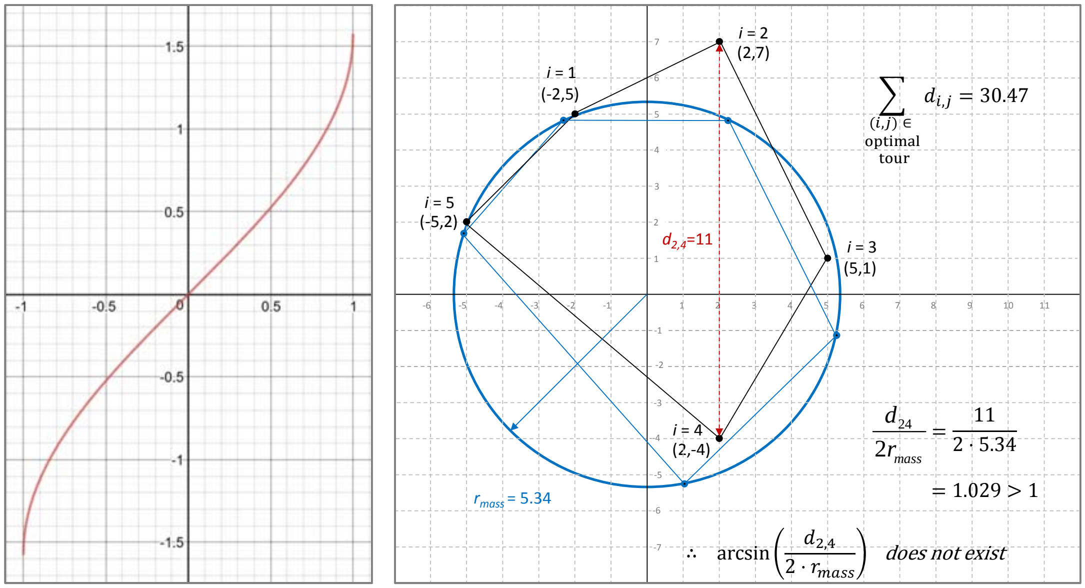


Figure 20. Left: $y = arcsin\,(x)$. Source: Desmos (2017);
Right: A Non-Optimal Arc.

2. Using the Air Assets Endurance as an Upper Bound

This property can be used in any tour, where the total distance is limited by some endurance. In this thesis, distances between COIs are known parameters at any time, an air asset's speed is constant, and its endurance is limited. Therefore, the total distance is bounded by an air asset's endurance.

If, $$\sum_{\substack{(i,j) \text{ s.t.} \\ (i,j) \in \text{ optimal} \\ \text{tour}}} \arcsin\left(\frac{d_{ij}}{2 \cdot r_{mass}}\right) = \pi \ ,$$

and $$2\pi \ r_{mass} \geq \sum_{\substack{(i,j) \text{ s.t.} \\ (i,j) \in \text{ optimal} \\ \text{tour}}} d_{ij} \qquad \Rightarrow \qquad r_{mass} \geq \frac{1}{2\pi} \sum_{\substack{(i,j) \text{ s.t.} \\ (i,j) \in \text{ optimal} \\ \text{tour}}} d_{ij} \ ,$$

then, $$\sum_{\substack{(i,j) \text{ s.t.} \\ (i,j) \in \text{ optimal} \\ \text{tour}}} \arcsin\left(\frac{\pi \cdot d_{ij}}{\displaystyle\sum_{\substack{(i,j) \text{ s.t.} \\ (i,j) \in \text{ optimal} \\ \text{tour}}} d_{ij}}\right) \geq \pi$$

Then, there exists a parameter δ such that:

$$\sum_{\substack{(i,j) \text{ s.t.} \\ (i,j) \in \text{optimal} \\ \text{tour}}} \arcsin\left(\frac{\pi \cdot d_{ij}}{\sum_{\substack{(i,j) \text{ s.t.} \\ (i,j) \in \text{optimal} \\ \text{tour}}} d_{ij}} \right) \le \pi + \delta \tag{4.8}$$

Since this thesis refers to air assets moving at a constant speed, we assume that the time $\tau_{i,j}$ to fly from COI i to COI j is proportional to the distance, and since the air asset has a limited endurance $\tau_{\max}$, we may write the following constraint (with δ big enough):

$$\sum_{i,j} \arcsin\left(\frac{\pi \cdot \tau_{ij}}{\tau_{\max}} \right) \cdot X_{ij} \le \pi + \delta \tag{4.9}$$

The ORCA VT formulation depends on the maximum endurance of each air asset type and a moving mother warship. If we consider a scenario where an air asset flies a long arc ahead of the mother warship, which increases speed to retake it, we might include the time that the air asset takes to fly from the initial position of the mother warship (1) to its final position $\tau_{1,w,h}$, ($w \in W_h$). This slightly increases the denominator and it is a more conservative approach, using δ to adjust the proximity to π. Hence, the heuristic constraints for ORCA VT are:

$$\sum_{i,j} \arcsin\left(\frac{\pi \cdot \tau_{i,j,h}}{\tau_{\max} + \tau_{1,w,h}} \right) \cdot X_{i,j,h} \le \pi + \delta \qquad \forall h \tag{4.10}$$

Letting $a_{i,j,h} = \arcsin\left(\frac{\pi \cdot \tau_{i,j,h}}{\tau_{\max} + \tau_{1,w,h}} \right)$, we get

$$\sum_{i,j} a_{i,j,h} \cdot X_{i,j,h} \le \pi + \delta \qquad \forall h \tag{4.11}$$

In constraint set (4.11), we consider that the air asset is moving following a circumference of length $\tau_{\max} + \tau_{1,w,h}$. If the time $\tau_{i,j,h}$ is small in relation to the

endurance, using constraints (4.11) is similar (but not identical) to using a constraint adding times $\tau_{i,j,h}$ (4.12).

$$\sum_{i,j} \tau_{i,j,h} \cdot X_{i,j,h} \leq air\ asset\ endurance\,, \qquad \forall h \qquad (4.12)$$

In constraint set (4.11), we are adding parameters that follow a nonlinear distribution (using the arcsine function), while simply adding the times $\tau_{i,j,h}$ (4.12) follows a linear one (all speeds are constant). This difference provides added restrictions as time increases (see next section).

3. Heuristic Limitations

Figure 21 shows data for all 80 COIs and a subset of 20 COIs. For 80 COIs, we see a bigger dispersion of COIs. COI # 2 in its last position is outlined in white in Figure 21. If we try to visit it coming from the opposite side, the time is substantial in relation to the air asset’s endurance and is eliminated by constraint set (4.11). It is unlikely (but not impossible given COI weighting) to obtain an optimal solution that includes this kind of arc. For example, if we only have two high priority COIs, and they are in opposite extreme positions from the mother warship, we may require such arcs for an optimal route.

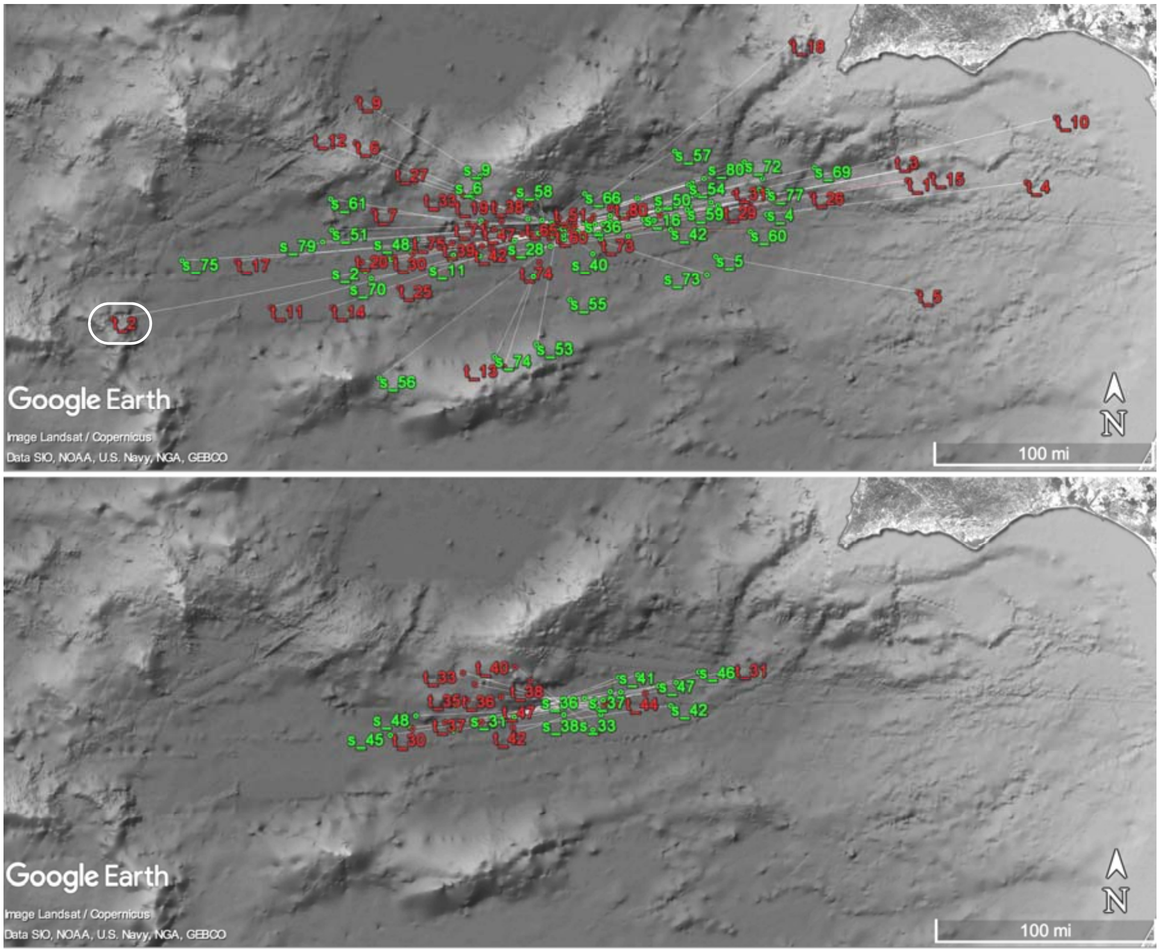


Figure 21. Set of 80 COIs (top) and Set of 20 COIs (bottom). Adapted from Google Earth (2016).

Figures 22 and 23 show the distributions of the arcsines $a_{i,j,h}$ and the times $\tau_{i,j,h}$. In both figures, we normalize the time value of each arc $\tau_{i,j,h}$ by using a factor (π / $\tau_{\max}$) to facilitate the comparison with the arcsine distribution. Whenever the value of the quotient $\pi\,\tau_{i,j,h}$ / ($\tau_{\max}$ + $\tau_{1,w,h}$) is greater than 1, we assign to $a_{i,j,h}$ the value of 4, to denote that the arcsine function would not exist, and to ensure that this arc does not satisfy constraint set (4.11). We use the letters A, B, C, and D to denote ranges of the distributions that we use to count the number of arcs eliminated and help in the discussion. The ranges of interest are B and C because this is where constraint sets 4.11 and 4.12 differ.

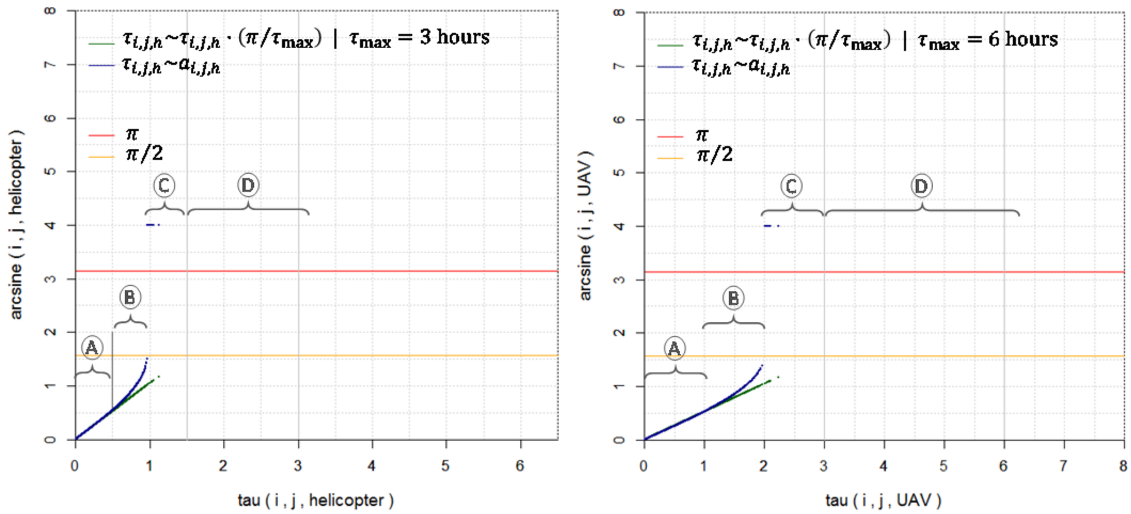


Figure 22. Data Distribution Plots (20 COIs and 10 Time Windows).

For 20 COIs (Figure 22), we see a small number of arcs in ranges B and C. By inspection, the difference between the times $\tau_{i,j,h}$ and the values of the arcsines $a_{i,j,h}$ are not significant, especially in range C, because there are no long arcs in relation to the air assets' endurance.

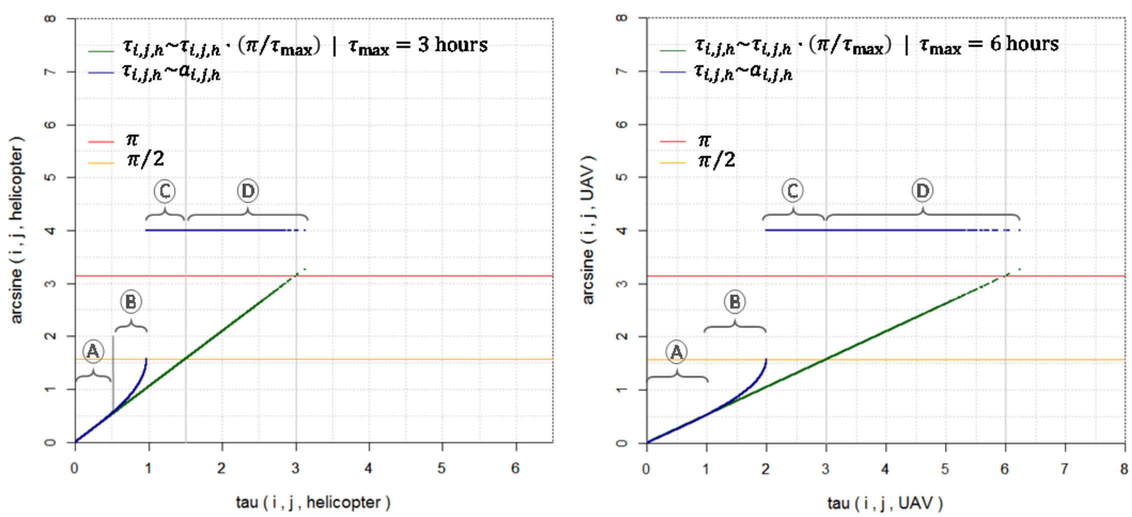


Figure 23. Data Distribution Plots (80 COIs and 10 Time Windows).

For 80 COIs (Figure 23), we see that the number of arcs that we eliminate (D) is significant. We would also eliminate these arcs by using a time constraint

set (4.12). However, using the heuristic, we eliminate most of the longest feasible arcs (C) and we penalize the use of the arcs in B. In Tables 3 and 4, we summarize all data for Figures 22 and 23.

Table 3. Data Distribution Summary (20 COIs and 10 Time Windows).

Asset	Helicopter			UAVs		
Type	range ($\tau_{i,j,h}$)	# arcs	%	range ($\tau_{i,j,h}$)	# arcs	%
A	0.00-0.43	39,280	89.07 *	0.00-1.12	42,354	96.04 *
B	0.43-0.96	4,800	10.88 *	1.12-1.99	1,732	3.92 *
C	0.96-1.11	20	0.04 *	1.99-2.99	14	0.03 *
ABC	0.00-1.11	44,100	100 **	0.00-2.99	44,100	100 **
D	-	0	0 **	2.99-6.24	0	0 **
ABCD	0.00-1.11	44,100	-	0.00-6.29	44,100	-

Table 4. Data Distribution Summary (80 COIs and 10 Time Windows).

Asset	Helicopter			UAVs		
Type	range ($\tau_{i,j,h}$)	# arcs	%	range ($\tau_{i,j,h}$)	# arcs	%
A	0.00-0.43	339,620	52.98 *	0.00-1.12	420,860	65.66 *
B	0.43-0.96	234,934	36.65 *	1.12-1.99	159,994	24.96 *
C	0.96-1.49	66,406	10.36 *	1.99-2.99	66,406	9.37 *
ABC	0.00-1.49	640,960	97.69 **	0.00-2.99	640,960	97.69 **
D	1.49-3.12	15,140	2.30 **	2.99-6.24	15,140	2.30 **
ABCD	0.00-3.12	656,100	-	0.00-6.29	656,100	-

A: Included with or without heuristic

B: Higher value with the heuristic

C: Arcs eliminated with the heuristic

D: Infeasible arcs (flight time is greater than half of the endurance)

* Percentage of arcs included in A, B, and C

** Percentage of all arcs

In the subset of 20 COIs (Table 3), the percentage of arcs where the heuristic can help is small for both helicopters and UAVs: very small in range C (almost zero), about 11% and 4% in range B, and zero in range D. For 80 COIs

(Table 4), about 10% of the arcs are in Section C, 30% in Section B, and just over 2% in Section D.

If there is a big number of nodes, such as the number of COIs multiplied by the number of time windows, eliminating arcs helps to obtain a solution faster. For 80 COIs and 20 time windows, the number of constraints in (3.6) is 10,497,600 ($81^2 \cdot 20^2 \cdot 4$), which is a challenging number. Eliminating 10% of the arcs (Section C) plus a nonlinear penalty in 30% of the arcs (Section B) helps to obtain a solution. If COIs were more dispersed, and the visiting routes were more challenging for the air assets because transition times are longer, then, by using this heuristic, we would eliminate a greater number of arcs because there would be more arcs in Sections B, C and D. If we pursue a good solution for a big problem instance in a reasonable time limit, we may want to include this heuristic as a decision between having this or having no solution at all.

VI. PROBLEM IMPLEMENTATION AND RESULTS

A. INTRODUCTION

This chapter shows ORCA implementations of data described in Chapter IV and compares results for both ORCA ILPs. We use a 2.5 GHz Intel(R) Core(TM) i5-6300U Microsoft Surface running Windows 10 PRO. We use Python 2.7.13 to prepare data. We employ GAMS 24.8.2 (2017) with the CPLEX solver version 12.7.0.0 (2017) to generate and solve ILP instances. We use Google Earth (2017) to show some routing graph solution. We use R version 3.3.2 (2016) to plot time distributions.

For both the ORCA VT and TI ILP formulations, we run the following tests (Table 5) using four air assets (1 SH60 plus 3 UAVs), different subsets of COIs, and different numbers of time windows (ORCA VT) or time steps (ORCA TI). We consider a typical scenario to be between 20 to 40 COIs, and 80 COIs to be a maximum that would rarely be encountered.

Table 5. Test Structure.

# COIs	ORCA VT # time windows	ORCA TI # time steps
20	5,10,15,20	5,10,15,20
40	5,10,15,20	5,10,15,20
80	5,10,15,20	5,10,15,20

20 COIs subset includes COIs from number 30 to number 49 in Table 2.

40 COIs subset includes COIs from number 20 to number 59 in Table 2.

80 COIs set is the complete Table 2.

The larger the number of time windows, the more precise the route will be. This number in the first formulation is limited by a constraint (3.6) which is $O(n^2)$, where n is the product of the number of COIs and the number of time windows. Instances using the second formulation have a similar number of constraints but

a smaller number of feasible arcs, although they require a larger number of time steps to be effective. Balancing the number of COIs and the number of time windows is a critical requirement to obtain acceptable solutions in a reasonable amount of time. First, we show results for both formulations as presented in Chapter III. Thereafter, and to speed solution time, we heuristically solve both formulations for one air asset at a time. Finally, we also use the heuristic arc elimination (Chapter V) in addition to solving for one air asset at a time only for the ORCA VT formulation.

B. RESULTS WITHOUT HEURISTICS

We show results of the ORCA VT and TI formulations (Tables 6 and 7) without heuristic additions and with a runtime limit of 30 minutes (we consider 30 minutes a reasonable time to get ready onboard while preparing to execute flying operations). For example, the first line in Table 6 shows a solution to the ORCA VT program with the set of 20 COIs with 5 time windows (105 nodes). The time limit (reslim) to solve the problem is 1,800 seconds, although the solver takes only 1,307.6 seconds to reach a relative gap (best integer solution quality) of 0.26% for an optimality stopping condition of 0.5% (optcr). The solution shows that the helicopter visits 20 COIs within its endurance and we use no UAVs, obtaining a total score objective function value of 2,991.9. All other rows are similar except when displaying the number of COIs visited by each UAV. This is recorded as a + and the number under the “# COIs visited” column.

Table 6. ORCA VT Program Outcomes.

# COIs	# TW	Runtime (seconds)	Rel. gap (%)	optcr (%)	reslim (secs)	# COIs visited	Total COIs Visited	Objective Function Value
20	5	1,307.6	0.26	0.5	1,800	20	20	2,991.9
	10	1,800.0	0.66	0.5	1,800	20	20	2,979.8
	15	1,800.1	0.96	0.5	1,800	20	20	2,967.8
	20	1,800.1	6.24	0.5	1,800	17+2+1	20	2,805.7
40	5	1,800.3	1.82	1.0	1,800	31+5+3+1	40	26,114.4
	10	1,800.1	10.56	1.0	1,800	14+11+13	38	23,788.8
	15	1,800.1	43.96	1.0	1,800	15+1+10+5	31	14,904.3
	20	1,800.3	51.40	1.0	1,800	5+5+1+8	19	12,900.7
80	5	1,801.1	79.24	1.0	1,800	5+1+1+1	8	10,645.9
	10	1,800.1	99.33	1.0	1,800	1+1+1+1	4	342.1
	15	1,800.3	99.93	1.0	1,800	1+1+1+1	4	1,231.8
	20	no solution	-	1.0	1,800	-	-	-

Table 7. ORCA TI Program Outcomes.

# COIs	# time steps	Runtime (Secs)	Rel. gap (%)	optcr (%)	reslim (secs)	# COIs visited	Total COIs Visited	Obj. Funct. Value
20	5	0.28	0.00	0.5	1,800	1+3+3+3	10	1,700.2
	10	0.67	0.00	0.5	1,800	3+4+5+8	20	2,300.6
	15	3.67	0.00	0.5	1,800	6+4+10	20	2,450.2
	20	4.28	0.00	0.5	1,800	8+1+11	20	2,549.6
40	5	0.42	0.00	1.0	1,800	1+3+3+3	10	17,000.2
	10	4.27	0.00	1.0	1,800	3+8+8+8	27	23,350.8
	15	37.9	0.00	1.0	1,800	6+10+12+12	40	25,051.6
	20	77.7	0.00	1.0	1,800	8+10+6+16	40	25,151.6
80	5	1.39	0.00	1.0	1,800	1+3+3+3	10	27,350.2
	10	25.5	0.00	1.0	1,800	3+8+8+8	27	38,200.8
	15	398.4	0.00	1.0	1,800	6+13+13+13	45	45,102.1
	20	no solution	-	1.0	1,800	-	-	-

Table 7, shows results using the ORCA TI formulation following the same format as Table 6. Comparing solutions from Tables 6 and 7, we see that the ORCA TI formulation is much faster in all the subsets of COIs but the quality of its solutions is directly impacted by the number of time windows. For 20 COIs, we

see that the ORCA VT formulation uses one air asset with 5, 10, and 15 time windows and achieves a greater reward for using the helicopter, while the TI needs more than one air asset.

For 80 COIs, the VT formulation fails even if we increase the runtime to one hour (Table 8), while the ORCA TI is limited by the number of time windows.

Table 8. ORCA VT-TI Outcomes Comparison (Increased Runtime).

# COIs	# TW /steps	Runtime (Secs)	Rel. gap (%)	optcr (%)	reslim (secs)	# COIs visited	Total COIs Visited	Obj. Funct. Value
80	20	no solution	-	1.0	3,600	-	-	-
	20	3,600.5	2.57	1.0	3,600	7+17+17+13	54	45,603.4

First row is for ORCA VT without heuristics.

Second row is for ORCA TI without heuristics.

C. RESULTS WITH LOOPING HEURISTIC

To speed solution time and improve results, we use a heuristic that solves for each air asset one at a time. We first solve for the helicopter, fix its solution, and then solve for one of the UAVs, fix its solution, and so on. Because the helicopter is the preferred air asset, we start with it. We limit the runtime for the helicopter to 1,200 seconds, and to 200 seconds for each one of the UAVs, to continue within a reasonable total time limit of 30 minutes. In Table 9, we show results for ORCA VT using this heuristic. For each time window scenario, we specify the results for each iteration (up to four iterations for the four air assets). For example, for 40 COIs (Table 9) and 15 time windows, the helicopter visits 24 COIs, obtaining a total score of 24,034.8, and the first UAV visits 16 COIs, adding only 1,216.0. Together they obtain a total score of 25,250.8.

In Table 10, we show results for ORCA TI using this looping heuristic.

Table 9. ORCA VT Program Outcomes with a Looping Heuristic.

# COIs	# TW	Runtime (seconds)	Rel. gap (%)	optcr (%)	reslim (secs)	# COIs visited	Total COIs Visited	Objective Function Value
20	15	1,142.8	0.96	1.0	1,200	20	20	2,967.8
	20	1,200.3	1.58	1.0	1,200	20	20	2,944.7
40	15	1,200.6 201.1	9.61 0.10	1.0 0.1	1,200 200	24 16	40	24,034.8 25,250.8
	20	1,201.1 1,75.8	5.64 0.09	1.0 0.1	1,200 200	26 14	40	25,087.7 25,739.7
80	15	1,202.4 204.1 206.0 207.9	99.61 99.93 99.92 99.91	1.0 0.1 0.1 0.1	1,200 200 200 200	1 1 1 1	4	196.8 243.8 291.8 338.8
	20	1,204.4 207.3 211.6 217.3	99.61 99.95 99.94 99.93	1.0 0.1 0.1 0.1	1,200 200 200 200	1 1 1 1	4	198.7 242.2 285.2 333.7

Table 10. ORCA TI Program Outcomes with a Looping Heuristic.

# COIs	# TW	Runtime (seconds)	Rel. gap (%)	optcr (%)	reslim (secs)	# COIs visited	Total COIs Visited	Objective Function Value
20	15	0.1 1.1 0.1 0.2	0.00 0.00 0.00 0.00	1.0 0.1 0.1 0.1	1,200 200 200 200	6 13 1 -	20	1,598.9 2,300.0 2,400.1 2,450.2
	20	0.2 0.5 0.3 0.5	0.00 0.00 0.00 0.00	1.0 0.1 0.1 0.1	1,200 200 200 200	8 12 - -	20	1,798.2 2,449.5 2,499.6 2,549.7
40	15	0.4 1.7 1.1 0.9	0.00 0.00 0.00 0.00	1.0 0.1 0.1 0.1	1,200 200 200 200	6 13 13 8	40	23,198.9 23,900.0 24,601.0 25,051.7
	20	1.62 25.9 3.2 1.78	0.00 0.00 0.00 0.00	1.0 0.1 0.1 0.1	1,200 200 200 200	8 18 14 -	40	23,398.2 24,350.1 25,101.6 25,151.7
80	15	2.7 8.1 6.0 8.4	0.00 0.00 0.00 0.00	1.0 0.1 0.1 0.1	1,200 200 200 200	6 13 13 13	45	42,098.9 43,700.0 44,401.0 45,102.1
	20	10.2 38.8 15.6 10.7	0.00 0.00 0.00 0.00	1.0 0.1 0.1 0.1	1,200 200 200 200	8 18 18 17	61	43,198.2 44,600.1 45,552.0 46,453.8

We see that using a looping heuristic (and fixing variables) improves both formulations by decreasing the runtime and reducing the number of air assets employed in the solution. However, ORCA VT is still slow in relation to the TI formulation, although it still wins in the score of the 20 time windows and uses only one air asset. In the 40 COIs subset, ORCA TI uses one or two more air assets than ORCA VT, although TI performs reasonably well in the set of 80 COIs (less than two minutes in the last instance), while the ORCA VT always terminates due to a time limit.

D. RESULTS WITH LOOPING HEURISTIC AND ARC ELIMINATION

To speed solution time, we consider the arc elimination heuristic combined with the looping heuristic from the previous section. In the subsets of 20 and 40 COIs, the looping heuristic produces a solution in less than half an hour. Hence, here we focus in the set of 80 COIs and a longer runtime of two hours. Since ORCA TI is limited by the number of time steps and it obtains quickly for all instances, we focus only on ORCA VT with 15 and 20 time windows (Table 11).

Table 11. ORCA VT with a Looping Heuristic and Arcs Elimination.

# COIs	# TW	Runtime (secs)	Rel. gap (%)	optcr (%)	reslim (secs)	# COIs visited	Total COIs Visited	Objective Function Value
80	15	3,604.5	34.18	1.0	3,600	20	75	33,469.8
		1,206.3	4.26	0.1	1,200	23		40,404.1
		1,207.5	1.82	0.1	1,200	21		41,392.1
		1,209.5	0.26	0.1	1,200	11		41,917.1
	20	3,608.9	13.82	1.0	3,600	25	77	43,823.9
		1,211.8	3.33	0.1	1,200	23		45,780.9
		1,216.6	1.39	0.1	1,200	18		46,588.4
		557.8	0.08	0.1	1,200	11		47,082.9

Using this combination of heuristics, ORCA VT obtains the highest reward visiting 77 COIs. It performs well in both time window scenarios and even better in the case with more time windows, and δ equal to 0.00001. In Tables 12 and

13, we show equivalent experiments with the same runtime (2 hours) and the same number of time windows.

Table 12. ORCA VT with a Looping Heuristic Only (Increased Runtime).

# COIs	# TW	Runtime (secs)	Rel. gap (%)	optcr (%)	reslim (secs)	# COIs visited	Total COIs Visited	Objective Function Value
80	15	3,602.4 1,204.1 1,207.0 1,208.2	99.61 58.53 10.45 5.01	1.0 0.1 0.1 0.1	3,600 1,200 1,200 1,200	1 12 23 19	55	196.8 10,667.4 23,005.8 24,344.8
	20	3,604.3 1,207.5 1,211.6 1,218.8	97.46 50.63 7.43 3.91	1.0 0.1 0.1 0.1	3,600 1,200 1,200 1,200	4 18 15 21	58	1,299.9 12,938.4 24,193.4 24,948.7

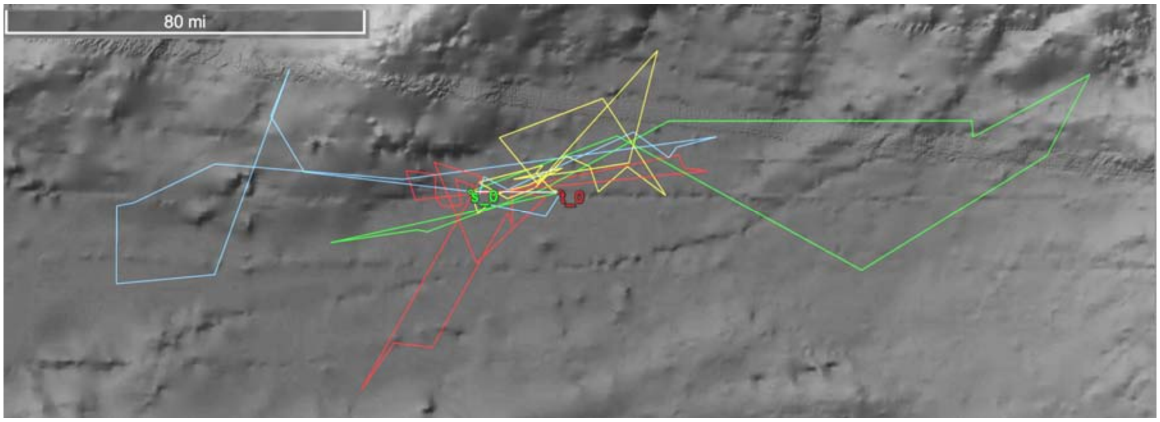
With only the looping heuristic and increased runtime (Table 12), we see the value of using the arc elimination constraint set. In the first loop with 20 time windows, the helicopter only visits 4 COIs with the looping heuristic only compared with 25 COIs when using both the looping heuristic plus the arc elimination constraint set.

Without using any heuristic with 20 time windows (Table 13), no solution is found even when given two hours.

Table 13. ORCA VT Outcomes (Increased Runtime).

# COIs	# TW	Runtime (secs)	Rel. gap (%)	optcr (%)	reslim (secs)	# COIs visited	Total COIs Visited	Objective Function Value
80	15	7,210.0	57.76	0.5	7,200	5+1+7+1	14	21,666.8
	20	no solution	-	0.5	7,200	-	-	-

In Figure 24, we show a plot of the routes corresponding to the second row of Table 11 (visiting 77 COIs).



Red, yellow, blue, and green routes correspond to the helicopter, and UAVs 1, 2 and 3.

Figure 24. Routes of Four Air Assets Visiting 77 COIs.

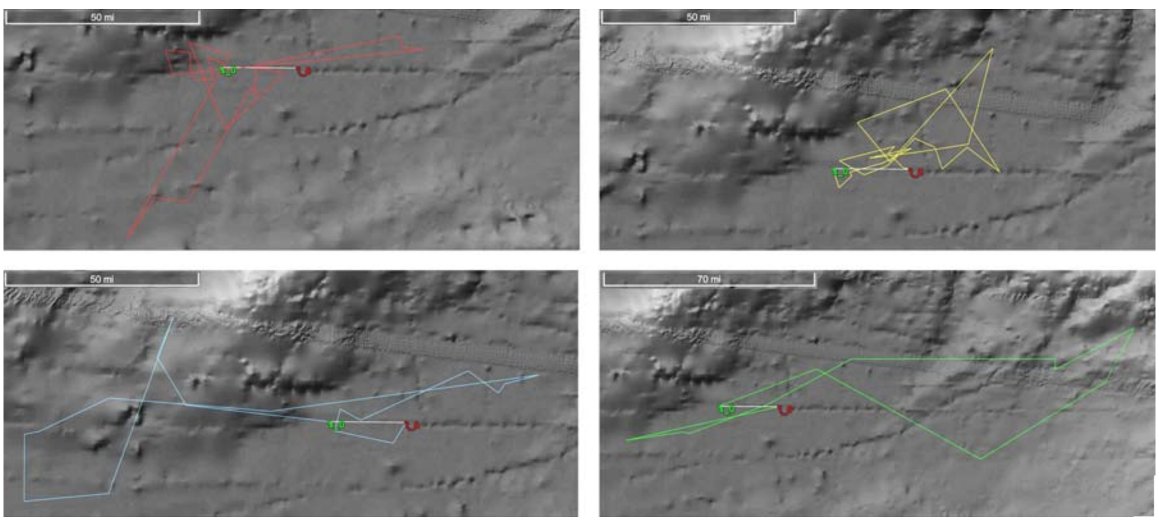


Figure 25. Single Routes of Four Air Assets Visiting 77 COIs.

VII. CONCLUSIONS AND OPPORTUNITIES

A. SUMMARY

Currently, the Tactical Action Officer (TAO) and the Air Asset or Helicopter Controller (HCO) route air assets based on their best understanding of the surface tactical picture without any decision aids. We need a tool to help them, and this thesis presents and tests the ORCA problem as a prototype for such a tool. Two different formulations of the ORCA problem recommend routes for visiting prioritized COIs at sea. ORCA TI is faster but, at times, requires more air assets to visit the same number of COIs. ORCA VT is generally slower but performs better with a small number of time windows and provides the best results when given more time.

We recommend using ORCA VT if the number of COIs is small (e.g., 20 COIs), there is a reasonable time to find an optimal route (e.g., 30 minutes), and we want to employ the smallest possible number of air assets. We recommend using ORCA TI if the number of COIs is small (e.g., 20 COIs), the time available to find an optimal route is short (e.g., less than 2 minutes), and minimizing the number of employed air assets is of less concern.

When the number of COIs is intermediate (e.g., 40 COIs), the looping heuristic is effective in both ORCA VT and TI. Again, ORCA VT uses fewer air assets but takes more time to obtain a solution than ORCA TI. Both formulations perform well within a time limit of 30 minutes.

When the number of COIs is big (e.g., 80 COIs) and there is a long time to plan air asset routes, ORCA VT with both heuristics (looping and arc elimination) is the best option. In less than two hours of runtime, ORCA VT obtains routes that visit as many as 77 COIs. If we need a quick solution, we recommend using ORCA TI. This formulation defines routes to visit up to 61 COIs in less than 1 minute by using just the looping heuristic.

B. FUTURE OPPORTUNITIES

Many extensions of the ORCA formulations may prove useful. First, we suggest including modifications to change initial positions of air assets, allowing dynamic retasking of air assets while already flying. Second, we may look for other heuristics that improve results and obtain faster solutions. Here, some deeper research in the arc elimination heuristic may be worthwhile, including testing more aggressive deletion rules.

Routing scenarios at sea vary greatly. ORCA is required to manage sets of COIs that differ widely in number, speeds, and dispersion. It would be good to test the effectiveness of ORCA solutions by comparing them to real-time routing scenarios aboard. More testing under different scenarios may help find other useful heuristics. In addition:

- Including ORCA within a combat system aboard warships is the most interesting future opportunity. We may require algorithms to check positions of COIs to correct the variability produced by small course and speed changes, as well as the influence of wind. This would allow air asset routes to be adjusted based on updated information.

- Planning multiple coordinated mother warships to identify COIs in a larger area (e.g., the Gulf of Aden) is a natural extension. If ships are sharing information using link data nets, we may have enough time to calculate multiple routes for multiple air assets leaving from multiple warships by using shared data from other warships, air units, satellite, and shore stations.

- The ORCA ILPs could be applied also for routing small patrol boats. We could integrate them on radar consoles. Operators could calculate best navigation paths to identify surface contacts by using a single or a coordinated group of multiple patrol boats.

- Another extension would be producing a sequential weapons employment plan. We may adapt ORCA to manage weapon systems instead of air assets, rewarding the destruction of threats instead of visiting COIs, while we have threats that vary in risk instead of COIs that vary in position. We should find how to define the risk variation as a function of time, and the transit time between threats for each weapon system, including both of them as data for the problem.

- GAMS (2017) with the CPLEX solver (2017) obtains good solutions, but there are free software opportunities like Pyomo (2017) for Python (2017) that may work to develop the ORCA ILPs. By doing so, we would not need to employ one programming language to prepare data and a different language to run the solver.

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